MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

EEL1176 – FIELD THEORY

(All sections / Groups)

17 OCTOBER 2016
9.00 a.m - 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 7 pages with 4 questions only.

2. Answer all **FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

(a) Three uniform vector fields are given by $\vec{A} = 2\hat{x} - \hat{z}$, $\vec{B} = 2\hat{x} - \hat{y} + 2\hat{z}$ and $\vec{C} = 2\hat{x} - 3\hat{y} + \hat{z}$. Determine the value of the following:

(i) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$. [2 marks]

(ii) $\vec{B} \bullet (\vec{C} \times \vec{A})$. [2 marks]

(iii) Angle between $\vec{B}$ and $\vec{C}$, θ_{BC}. [2 marks]

(iv) The component of $\vec{A}$ along $\vec{B}$. [2 marks]

(b) Given the Cartesian coordinates of Q = (−1, 3, 2) and vector $\vec{A} = (x)\hat{x} + (y+z)\hat{y}$. Determine the following:

(i) Point Q in cylindrical coordinates. [2 marks]

(ii) Vector $\vec{A}$ at Q in Cartesian coordinates system. [1 mark]

(iii) Vector $\vec{A}$ in cylindrical coordinates system. [3 marks]

(iv) Vector $\vec{A}$ at Q in cylindrical coordinates system. [1 mark]

(c) Given that a vector field $\vec{F} = (r\cos\phi)\hat{r} + (\sin\phi)\hat{\phi}$. Verify the Stokes's theorem over a quarter-circular disk with a radius, r = **3** in the first quadrant, as shown in **Figure Q1(c)**, by evaluating:

(i) $\oint_C \vec{F} \bullet d\vec{l}$. [5 marks]

(ii) $\int_S (\vec{\nabla} \times \vec{F}) \bullet d\vec{s}$. [5 marks]

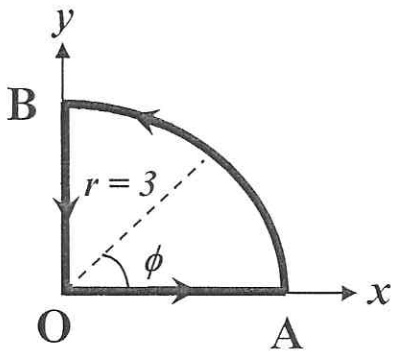

Figure Q1(c)

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Question 2

(a) **Coulomb's law** describes the electrostatic interaction between electrically charged particles. Point charges are placed at the corners of a square of size 4 m as shown in **Figure Q2(a)**. If $Q = 10\ \mu C$, calculate the following:

(i) Total electric force on a 10 nC charge located at (0, 0, 6). [8 marks]

(ii) Total electric field intensity, E at (0, 0, 6). [4 marks]

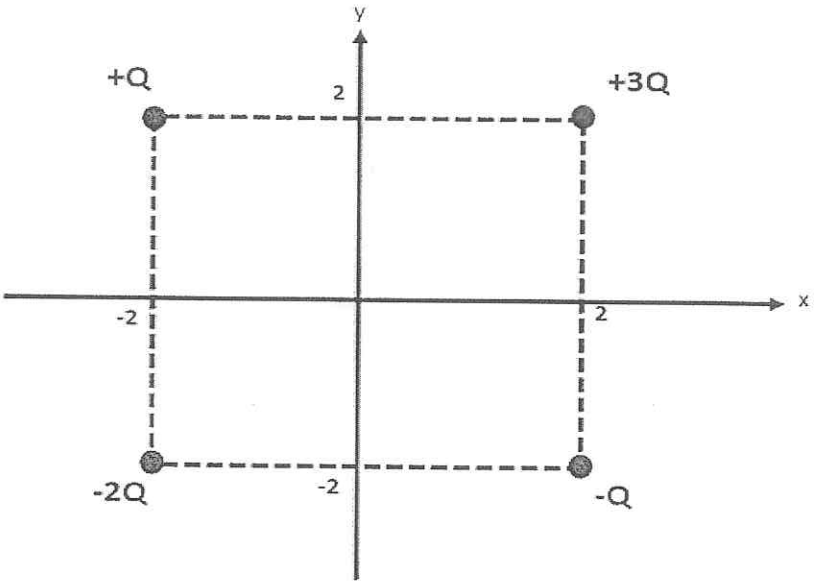

Figure Q2(a)

(b) Given the electric field $\bar{E} = \hat{x} + z^2\hat{y} + 2yz\hat{z}$ V/m, calculate the work done in carrying a 5 C charge from $(1, 2, -4)$ to $(3, -5, 6)$ in the electric field.

[5 marks]

(c) **Figure Q2c(i)** and **Figure Q2c(ii)** represent the largest cross sections of two spherical capacitors. Given a = 1 mm, b = 3 mm, c = 2 mm, ξ_{r1} = 2.5 and ξ_{r2} = 3.5, determine the capacitances of the two spherical capacitors.

[8 marks]

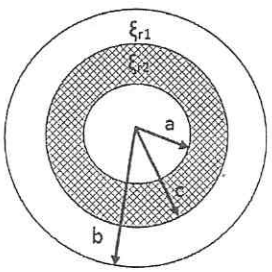

Figure Q2c (i)

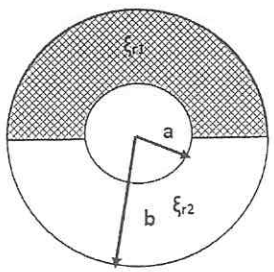

Figure Q2c (ii)

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Question 3

(a) A long coaxial cable is made by sandwiching an outer conductor and an inner conductor separated by an insulator layer. The radius of the inner conductor is a, and the inner and outer radii of the outer conductor are b and c respectively, refer to **Figure Q3(a)**. The current, I, flows along the positive z-direction in the inner conductor and returns through the outer conductor. Determine the magnetic field intensity, $\boldsymbol{H}$ for the following regions in the coaxial cable :

(i) $r < a$, [5 marks]

(ii) $a < r < b$, [5 marks]

(iii) $b < r < c$, [5 marks]

(iv) $r > c$. [5 marks]

(b) Plot the magnitude of $\boldsymbol{H}$ as a function of r, for all the regions mentioned in Part (a). [5 marks]

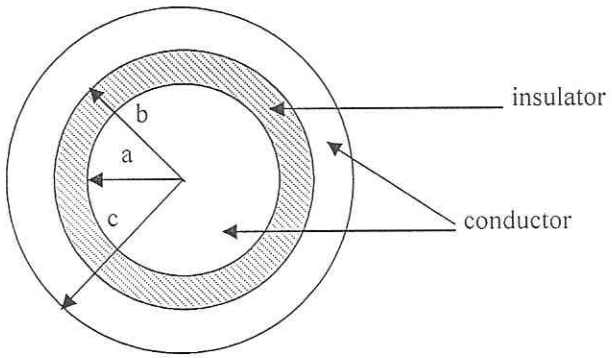

Figure Q3(a)

Question 4

In the magnetic circuit shown in **Figure Q4(a)**, the coil at the left arm with 1000 turns is supply with 4 A. The relative permeability of the core is 2500. The cross-sectional area of the magnetic core is a constant 40 cm^2. Assume no fringing effect and no flux leakage. Calculate:

(i) the reluctance in left arm and air gap. [6 marks]

(ii) the magnetomotive force produce by the coil on the left arm [2 marks]

(iii) the magnetic flux in the right arms. Given the flux in the air gap is 4×10^{-3} Wb from top to bottom. [7 marks]

(iv) the magnetomotive force, F in the center arm. [4 marks]

(v) Magnetic flux density on right arm. [2 marks]

(vi) Magnetic flux intensity on left arm. [4 marks]

Continued ...

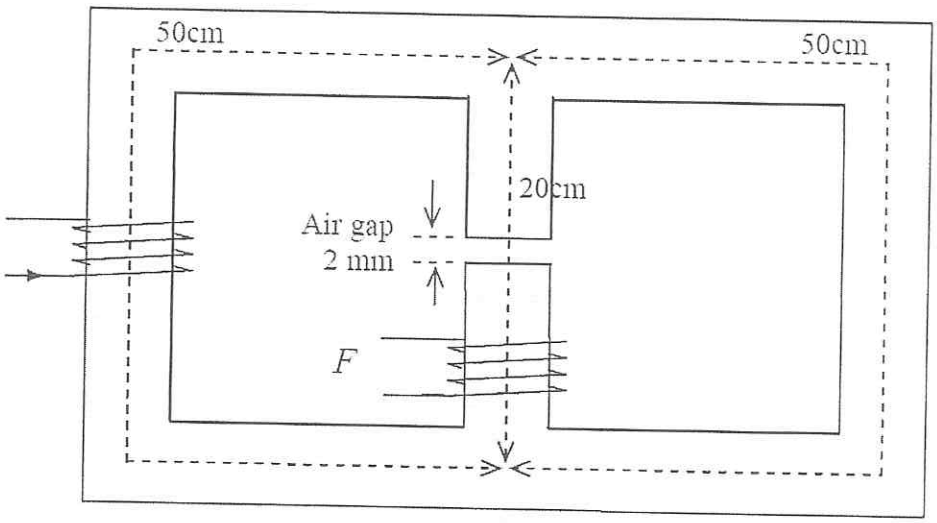

Figure Q4(a)

End of Paper

Appendix A: Physical Constants, Vector and Coordinate Transformations

Elementary charge	e	1.60×10^{-19} C
Permittivity of free space	ε_0	8.85×10^{-12} F/m
Permeability of free space	μ_0	1.26×10^{-6} H/m $= 4\pi \times 10^{-7}$ H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}\, r\, d\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\bar{s}_x = \hat{x}\, dydz$ $d\bar{s}_y = \hat{y}\, dxdz$ $d\bar{s}_z = \hat{z}\, dxdy$	$d\bar{s}_r = \hat{r}\, rd\phi\, dz$ $d\bar{s}_\phi = \hat{\phi}\, dr\, dz$ $d\bar{s}_z = \hat{z}\, rdr\, d\phi$	$d\bar{s}_R = \hat{R}\, R^2 \sin\theta\, d\theta\, d\phi$ $d\bar{s}_\theta = \hat{\theta}\, R\sin\theta\, dR\, d\phi$ $d\bar{s}_\phi = \hat{\phi}\, R\, dR\, d\theta$
Differential volume	$(dx)(dy)(dz)$	$(dr)(r\, d\phi)(dz)$	$(dR)(R\, d\theta)(R\sin\theta\, d\phi)$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian & Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$

Spherical & Cartesian	$R=\sqrt{x^2+y^2+z^2}$ $\theta=\tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi=\tan^{-1}(y/x)$	$\hat{R}=\hat{x}\sin\theta\cos\phi+\hat{y}\sin\theta\sin\phi+\hat{z}\cos\theta$ $\hat{\theta}=\hat{x}\cos\theta\cos\phi+\hat{y}\cos\theta\sin\phi-\hat{z}\sin\theta$ $\hat{\phi}=-\hat{x}\sin\phi+\hat{y}\cos\phi$	$A_R=A_x\sin\theta\cos\phi+A_y\sin\theta\sin\phi+A_z\cos\theta$ $A_\theta=A_x\cos\theta\cos\phi+A_y\cos\theta\sin\phi-A_z\sin\theta$ $A_\phi=-A_x\sin\phi+A_y\cos\phi$
	$x=R\sin\theta\cos\phi$ $y=R\sin\theta\sin\phi$ $z=R\cos\theta$	$\hat{x}=\hat{R}\sin\theta\cos\phi+\hat{\theta}\cos\theta\cos\phi-\hat{\phi}\sin\phi$ $\hat{y}=\hat{R}\sin\theta\sin\phi+\hat{\theta}\cos\theta\sin\phi+\hat{\phi}\cos\phi$ $\hat{z}=\hat{R}\cos\theta-\hat{\theta}\sin\theta$	$A_x=A_R\sin\theta\cos\phi+A_\theta\cos\theta\cos\phi-A_\phi\sin\phi$ $A_y=A_R\sin\theta\sin\phi+A_\theta\cos\theta\sin\phi+A_\phi\cos\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$
Cylindrical & Spherical	$R=\sqrt{r^2+z^2}$ $\theta=\tan^{-1}(r/z)$ $\phi=\phi$	$\hat{R}=\hat{r}\sin\theta+\hat{z}\cos\theta$ $\hat{\theta}=\hat{r}\cos\theta-\hat{z}\sin\theta$ $\hat{\phi}=\hat{\phi}$	$A_R=A_r\sin\theta+A_z\cos\theta$ $A_\theta=A_r\cos\theta-A_z\sin\theta$ $A_\phi=A_\phi$
	$r=R\sin\theta$ $\phi=\phi$ $z=R\cos\theta$	$\hat{r}=\hat{R}\sin\theta+\hat{\theta}\cos\theta$ $\hat{\phi}=\hat{\phi}$ $\hat{z}=\hat{R}\cos\theta-\hat{\theta}\sin\theta$	$A_r=A_R\sin\theta+A_\theta\cos\theta$ $A_\phi=A_\phi$ $A_z=A_R\cos\theta-A_\theta\sin\theta$

Appendix B: Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V=\hat{x}\frac{\partial V}{\partial x}+\hat{y}\frac{\partial V}{\partial y}+\hat{z}\frac{\partial V}{\partial z}$$

$$\nabla\cdot\vec{A}=\frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{A}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}=\hat{x}\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)+\hat{y}\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)+\hat{z}\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2V=\frac{\partial^2V}{\partial x^2}+\frac{\partial^2V}{\partial y^2}+\frac{\partial^2V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V=\hat{r}\frac{\partial V}{\partial r}+\hat{\phi}\frac{1}{r}\frac{\partial V}{\partial\phi}+\hat{z}\frac{\partial V}{\partial z}$$

$$\nabla\cdot\vec{A}=\frac{1}{r}\frac{\partial}{\partial r}(rA_r)+\frac{1}{r}\frac{\partial A_\phi}{\partial\phi}+\frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{A}=\frac{1}{r}\begin{vmatrix}\hat{r} & \hat{\phi}r & \hat{z}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z}\\ A_r & rA_\phi & A_z\end{vmatrix}=\hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\phi}-\frac{\partial A_\phi}{\partial z}\right)+\hat{\phi}\left(\frac{\partial A_r}{\partial z}-\frac{\partial A_z}{\partial r}\right)+\hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\phi\right)-\frac{\partial A_r}{\partial\phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2}+\frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R}+\hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta}+\hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R)+\frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta)+\frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla\times\bar{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}\left(A_\phi\sin\theta\right)-\frac{\partial A_\theta}{\partial\phi}\right]+\hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi}-\frac{\partial}{\partial R}\left(RA_\phi\right)\right]+\hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}\left(RA_\theta\right)-\frac{\partial A_R}{\partial\theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)+\frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

Appendix C: Table of Integrals

$$\int \sin^2\theta d\theta = \frac{\theta}{2}-\frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta d\theta = \frac{\theta}{2}+\frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2-x^2}\,dx = -\frac{1}{3}\left(a^2-x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2+z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2+z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2+z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2+z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

End of Appendix